A Biomimetic Moving-Mesh Topology Optimization Method

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A biomimetic moving-mesh topology optimization method

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Abstract

Topology optimization has experienced rapid development over the past two decades and has been widely applied in fields such as aircraft structures, civil engineering, and transportation equipment. Common topology optimization methods, such as density-based methods and level set methods, focus on global variable optimization. These global optimization approaches often consume substantial computational resources and are not suitable for parallel optimization. In contrast, structures in nature evolve from a combination of numerous local optimization problems, where each cell unit adjusts on the basis of its perception of the surrounding environment, leading to the formation of biological structures. This paper proposes a novel heuristic topology optimization method, the biomimetic moving-mesh (BMM) method, inspired by biological cell growth and evolution. The BMM method uses the positions of mesh nodes as variables to simulate cellular expansion and contraction, thereby creating a new optimization approach. Compared with traditional topology optimization methods, the BMM method offers smoother meshes and is more suitable for handling large-scale parallel optimization problems.

Keywords: Biomimetic, Moving-mesh, Local optimization, Topology optimization.

1. Introduction

Over the past two decades, topology optimization (TO) has sparked widespread interest among engineers and researchers because of its powerful ability to determine optimal material layouts within a design domain for desired performance. Structural topology optimization is widely used in the fields of aircraft design [1], engineering architecture [2], vehicle architecture [3], mechanical parts [4], and metamaterial microstructure design [5]. There are many optimization objectives for structural design in various disciplines, such as acoustic [6], optical [7], force [8], electrical [9], magnetic [10], thermal [11], and other physical properties.

In 1988, Bendsøe and Kikuchi [12] introduced the homogenization method, which laid the foundation for generating optimized topologies in the design domain. This marked the beginning of significant developments in topology optimization. In recent years, various topology optimization methods have been developed, each with specific functions. Notable methods include the solid isotropic material with penalization (SIMP) method [13], the evolutionary structural optimization (ESO) method [14], the level set method (LSM) [15], and the moving morphable components (MMC) method [16]. These methods have been applied to solve various problems, including heat conduction [17], material design [18], and concurrent design [19].

One limitation of most traditional topology optimization methods is that their results are derived from a predefined structured grid, necessitating extensive postprocessing. During this phase, the finite element mesh must be repartitioned and recalculated, with the optimization effects subsequently revalidated. Although approaches such as the level set method and moving-morphable components (MMCs) [16] generate smooth boundaries, in practice, they achieve this by smoothing the edges of the initial grid and adjusting the stiffness matrix of the boundary elements. This can lead to computational outcomes that are not entirely accurate.

In topology optimization, determining the boundaries during the optimization process is a key challenge. For example, in methods such as the level set and MMC methods, the finite element mesh of the structure is separate from the geometric model description. Particularly in the LSM [20], when boundaries intersect the fixed finite element mesh, quadrilateral elements are bisected, which reduces the stiffness of the elements and fails to simultaneously meet the requirements for smooth geometric and finite element boundaries. Although boundary-adaptive refinement of the finite element mesh can increase boundary precision, this treatment significantly reduces the computational efficiency.

Existing topology optimization methods often fail to achieve smooth synchronization between finite element meshes and geometric descriptions. Most current approaches are based on a global perspective, focusing on calculating the overall descent direction, with limited attention given to local adaptive adjustment methods. If a topology optimization method could be developed that dynamically integrates the optimization process with the finite element mesh model, it would enable structures to satisfy both finite element and geometric boundary smoothness requirements. This advancement would facilitate the simultaneous integration of finite element analysis and CAD modeling, thus enhancing the overall design process. To achieve this, the structure of this article is organized as follows. Chapter 2 focuses on introducing the evolutionary background of biological structures derived from cellular development. Chapters 2.1 through 2.3 collectively introduce the biomimetic moving-mesh method, which integrates contour boundary-driven shape optimization, material flipping for topological transformations, and adaptive mesh adjustments to increase computational accuracy. Chapter 3 presents simple examples showing the principles of the BMM method. Chapters 4 and 5 demonstrate the application of this method to 2D and 3D classic topology optimization problems and further analyze its advantages over other traditional topology optimization methods. Chapter 6 presents the conclusions and a discussion.

2. A biologically inspired topology optimization method

Before we present the proposed topology optimization method, we first discuss certain aspects related to topology optimization.

Strength optimization vs. stiffness optimization: For strength optimization, there are many strength criteria, such as the von Mises stress, maximum tensile stress, maximum elongation line strain, and maximum shear stress, etc. The sensitivity information can also be calculated on the basis of these parameters. If the strain energy density is chosen as the strength criterion (although this is not a common strength criterion), the sensitivity calculation is consistent with the result of the stiffness optimization problem, so the stiffness topology optimization problem can be considered a special case of the strength topology optimization problem. Therefore, strength optimization is a more general optimization. To some extent, stiffness optimization is only a special case of strength optimization. Therefore, we believe that strength optimization warrants more research attention, as it is closely linked to the optimization of biological growth, which is inherently related to stress.

Global optimization vs. local optimization (or local evolution): Optimization methods are divided into global optimization and local optimization. Global optimization often requires finding a direction of descent, e.g., along a gradient. Here, the terms "global optimization" and "local optimization" refer not to the pursuit of global or local optimal solutions for nonconvex problems but rather to whether the optimization process requires global information or local information. Derivation is a time-consuming problem for optimization problems with a particularly large number of variables, and it consumes a large amount of computational resources to obtain the global gradient, which can be greatly accelerated if it can be split into many small local optimization problems. This is the reason why an algorithm more suitable for large-scale optimization can be inspired by a local biological evolution strategy.

Nature is a master of structural optimization. For example, the internal microstructure of the hornbill bird beak obtained very similar results to the gig voxel-scale wing topology optimization performed by Niels Aage [21] via a computer in 2017, which took several days to obtain the results via 8,000 CPU calculations, whereas the former results were only obtained by the organism's own adaptive growth and deformation to the environment. This shows that biological evolution is a very inspiring optimization method.

Furthermore, dynamic cell migration [22, 23] plays a crucial role in processes such as embryonic development, wound healing, and cancer cell metastasis. During dynamic migration, the boundaries of cell clusters expand or contract in response to external stimuli. These biological processes depend on factors such as cellular activities, environmental interactions, and functional requirements, which in turn shape cellular morphology, which is also the basis for bionics to construct structural forms. The expansion, contraction, fusion, and division of biological cells are governed by intrinsic critical thresholds that operate independently of overall control mechanisms.

As a parallel to this biological phenomenon, in the optimization model discussed in this paper, we analogize cell clusters to a finite element mesh, where the movement of cell boundaries is simulated through the dynamic adjustment of mesh nodes.

In this work, the biomimetic moving-mesh (BMM) topology optimization method is introduced within the framework of continuum medium mechanics, reflecting the biological logic of cells moving toward favorable conditions and away from unfavorable conditions. This method can be divided into three steps: adaptive movement of contour nodes, element material flipping, smoothing of contour meshes and uniform distribution of internal meshes.

2.1 Adaptive movement of contour nodes

When we drive the movement of a structure's boundary, the shape of the topological structure changes, analogous to how the boundaries of biological cell groups continuously expand and contract in response to environmental stimuli.

Determining the movement step length for each boundary node is crucial for the optimization process. The relationship between a node's average strain energy and its movement step length along the boundary normal can be expressed in the linear form $\boldsymbol{l} = \alpha(\boldsymbol{D} - \boldsymbol{D_0})$. $\boldsymbol{D}$ represents the sensitivity of the unit distance of node movement with respect to the objective function (which is stress in strength problems and strain energy in compliance problems). $\boldsymbol{D_0}$ (danger point) is a constant vector, which means that when it exceeds a certain value, the node expands outward, and when it falls below a certain value, the node contracts inward. This value is determined by the volume fraction constraint, and the specific calculation formula is provided later. $\boldsymbol{l}$ is the movement step length along the boundary normal direction. In this context, α acts as a scaling factor for the movement step length. Notably, the relationship between step size and sensitivity is not limited to a linear relationship but can also be quadratic, as can other functional forms. In this paper, the simplest linear relationship is used for illustration.

To ensure stability during the iterative process, it is advisable to avoid movements exceeding half the length of an element in a single iteration. Hence, the scaling factor α should be set to control the maximum single movement step length to approximately 0.1 ~ 0.5 unit lengths.

During the optimization process, constraints on the volume fraction of the structure must be considered to ensure that the volume fraction approaches the target value after each iteration. The value of $\boldsymbol{D_0}$ can be derived from Eq. (1),

$$\alpha(\boldsymbol{D} - \boldsymbol{D_0})\boldsymbol{A} = r\left(V^{\text{solid}} - \rho V\right) \tag{1}$$

which is related to maintaining the volume fraction. The method of approaching volume constraints refers to the bidirectional evolutionary structural optimization (BESO) method.

The variables in the formula are explained as follows:

$\boldsymbol{A}$ represents the sensitivity of the volume fraction, i.e., the change in the total volume fraction when each node moves a unit length in the normal direction.

r represents the evolutionary rate, indicating the speed at which the volume fraction converges toward the predefined target constraint, commonly established at 0.01.

V^{solid} is the volume of the solid material in the current structure.

ρ is the target volume fraction.

V is the total volume of the design domain.

This formula determines the step size to be applied to each boundary node during the current iteration, contingent upon the volumetric change in the solid material. This approach provides a direct and uncomplicated mechanism to facilitate the displacement of boundary nodes.

However, while this approach excels in guiding shape optimization, it does not address the challenges associated with topology transformations. These issues are explored and addressed below.

2.2 Element material flipping enables topology transformation

In the process of structural optimization, as grid nodes are repositioned, the area (or volume) of certain elements may decrease significantly, rendering these materials potentially superfluous. One approach to address this involves transitioning materials on the basis of a predetermined area threshold, for example, 0.2. In general, materials within the design domain can be classified into hard and soft categories to differentiate the presence or absence of material in a given space. The Young's modulus of hard materials can be set to $1\,(\mathrm{N/m^2})$, whereas the Young's modulus of soft materials can be set to $1\times10^{-9}(\mathrm{N/m^2})$. Here, hard phase materials are converted to soft phase materials when the area (or the volume) of an element drops below this threshold, and vice versa.

It is imperative to consider the impact of node movement on the quality of mesh elements to ensure computational accuracy in structural optimization. Specifically, when elements become concave quadrilaterals or develop excessive aspect ratios (e.g., 10:1), such configurations can significantly increase the error of the computational model. These scenarios may require interventions such as material flipping or other corrective measures to preserve the quality of the mesh, as depicted in Fig. 1. This emphasizes the critical need to maintain a high-quality mesh throughout the optimization process, thereby ensuring reliable and efficacious structural outcomes.

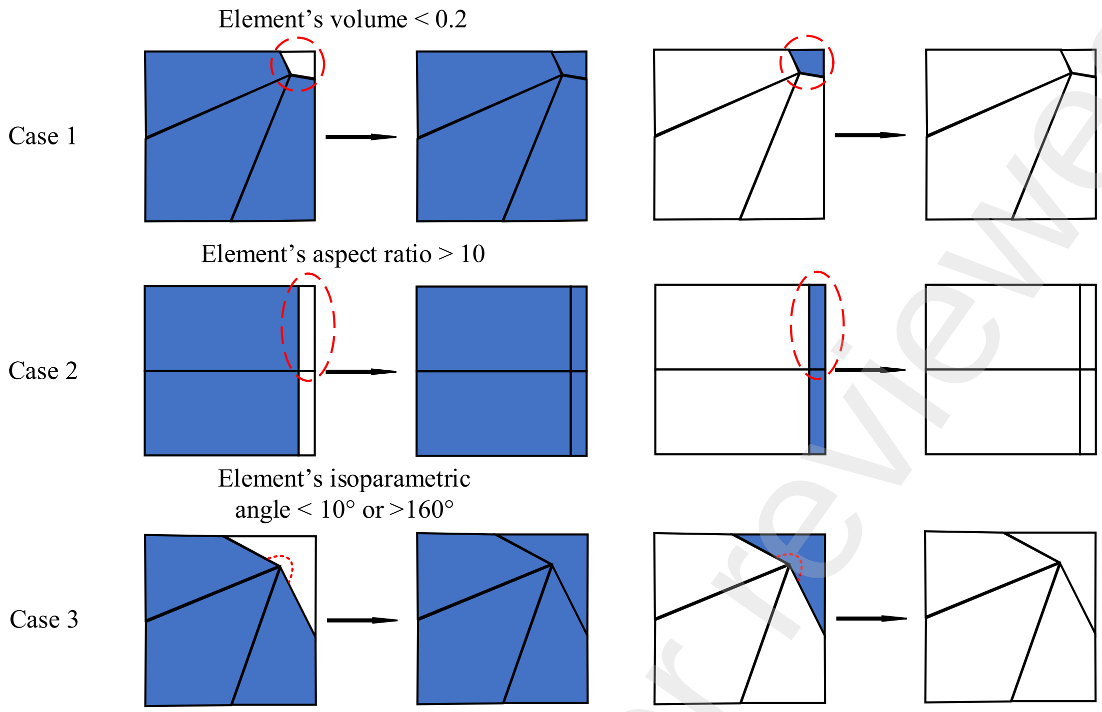


Fig. 1. Three scenarios that trigger material flipping.

2.3 Smoothing of contour meshes and uniform distribution of internal meshes

After adjusting boundary nodes and flipping materials in a structure, the quality of the internal mesh can be compromised. To rectify this and achieve a more uniform mesh, smoothing techniques are commonly applied to distorted grids. This mesh smoothing process aids in redistributing the grid nodes more evenly across the mesh, enhancing both the mesh quality and the computational accuracy. Importantly, this technique does not alter the overall shape of the structure or impact the integrity of the final results, ensuring that the structural performance remains unaffected.

The mesh smoothing process is depicted in Fig. 2, where the coordinates of each noncontour node are adjusted to the average coordinates of their four neighboring nodes. This technique can be mathematically represented by Eq. (2). By implementing this approach, the uniformity within the internal mesh is enhanced, which facilitates a smoother transition between the boundary and internal grids, effectively reducing discontinuities and abrupt changes.

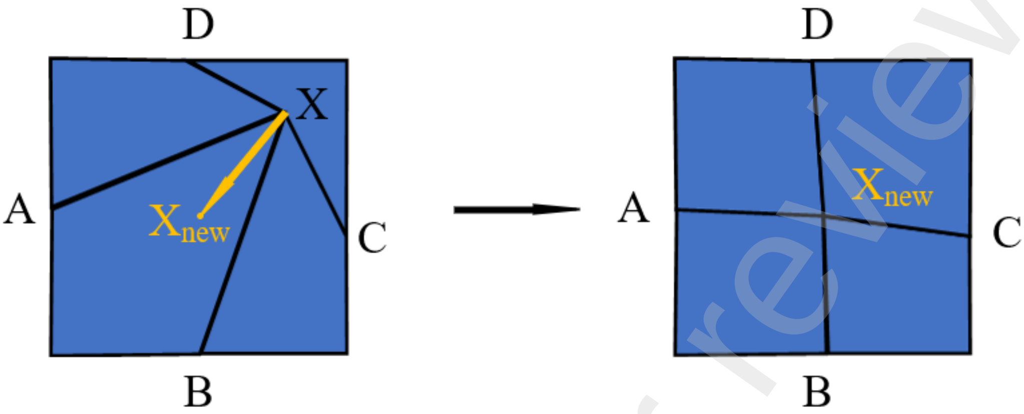


Fig. 2. Adaptive tuning of nodes inside structured grids.

$$X_{new}(x,y)=\frac{A(x,y)+B(x,y)+C(x,y)+D(x,y)}{4} \tag{2}$$

In structural optimization, meticulous management of contour nodes is essential for ensuring smooth boundaries. A specific method that capitalizes on the unique properties of contour nodes in determining their movement involves the following steps: initially, the vector defined by each contour node and the midpoint is computed between its left and right neighboring nodes $(\boldsymbol{X}_i^{\text{left}}+\boldsymbol{X}_i^{\text{right}})/2-\boldsymbol{X}_i$; subsequently, this vector is scaled by a factor β (e.g., 0.01) and added to the movement step of the contour nodes.

The rationale behind this methodology is to incorporate a dual-component strategy in each node's movement during the optimization cycles, as shown in Eq. (3):

$$\boldsymbol{v}_i=l_i\boldsymbol{n}_i+\beta\left(\frac{\boldsymbol{X}_i^{\text{left}}+\boldsymbol{X}_i^{\text{right}}}{2}-\boldsymbol{X}_i\right)=\alpha(D_i - D_{0i})\boldsymbol{n}_i+\beta\left(\frac{\boldsymbol{X}_i^{\text{left}}+\boldsymbol{X}_i^{\text{right}}}{2}-\boldsymbol{X}_i\right) \tag{3}$$

The first term $l_i\boldsymbol{n}_i$ stems from the sensitivity analysis, termed the adaptive vector, which is responsive to environmental changes. This represents the product of the outward normal vector $\boldsymbol{n}_i$ at contour node i and its step length l_i. The second term is the smoothing vector, which is specifically designed for contour nodes. This specialized approach for contour node movement ensures that the boundary edges become smoother.

The flowchart of the entire optimization algorithm is shown in Fig. 3.

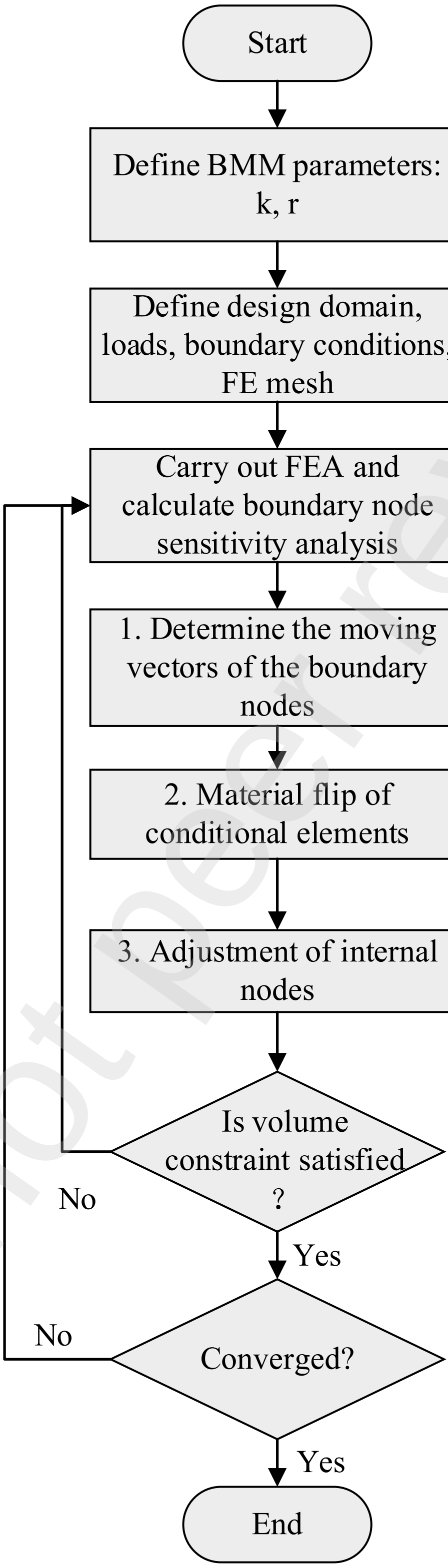


Fig. 3. Flowchart of the BMM topology optimization method.

3. Simple examples showing the principle of the BMM method

Considering a periodic structure with square holes subjected to uniform biaxial compression, Figure 4 illustrates the stress distribution. The shape of the holes is modified by moving the boundary nodes in normal directions, as indicated by the arrows in the diagram, with the step length of movement linearly related to the stress magnitude. After 40 iterations, the shape of the holes gradually transitions from square to circular, optimizing the boundary shape of the structure through environmental adaptation.

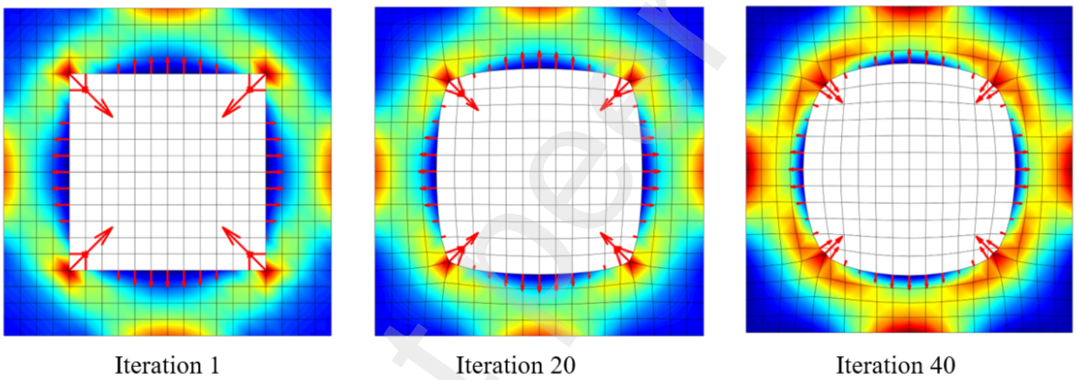


Fig 4. Adaptive movement of internal borehole nodes under uniform biaxial compression.

Consider another scenario involving a structure with two square holes: the bottom of the structure is fixed, whereas the top faces a uniform downward load. During the loading phase, the material located between the two holes might exhibit lower stress, prompting the contour boundaries to progressively converge. As the volume of material in the middle diminishes and reaches a critical threshold, a change in the material state occurs, enabling the two holes to merge into a single larger hole, thereby facilitating a topological transformation. Conversely, when the boundaries of hard material blocks intersect, the number of distinct holes may increase.

By identifying and adjusting unnecessary materials, this approach allows for automatic adjustment of the material topology during structural design optimization. Figure 5 illustrates this dynamic, showing how the merging of holes can result in profound changes in the structure's topology, optimizing both the mechanical properties and material efficiency. This strategy not only facilitates the merging of holes but also allows for their splitting (for instance, when two endpoints of a hole move closer and connect, resulting in one hole being divided into two). Moreover, the strategy can also introduce methods to accelerate hole formation, such as selecting the solid element with the lowest stress or strain energy density from the structure to be flipped to a void every certain number of iterations. At this point, a new hole emerges, continuing evolution along the boundaries of new holes. However, methods for accelerating hole formation are still under investigation and remain unstable. Currently, the approach of merging holes alone can yield very effective optimization results.

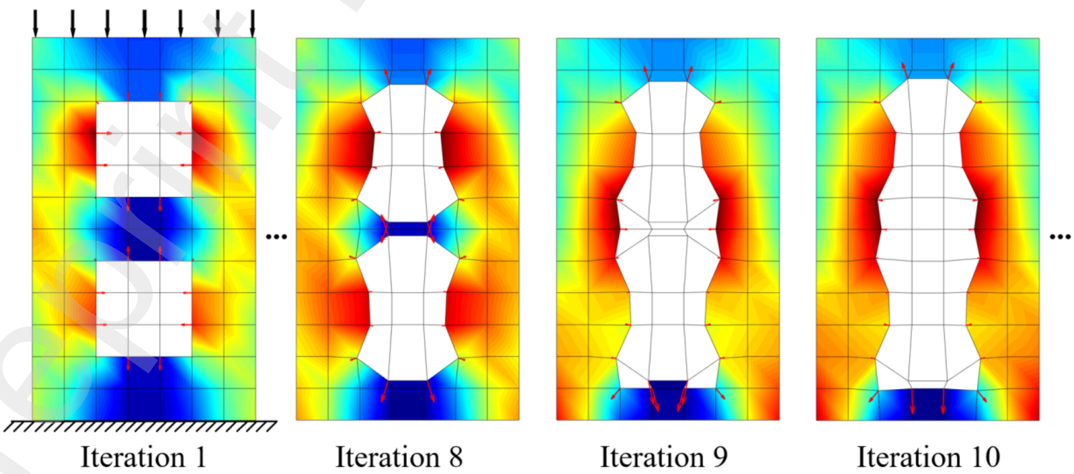


Fig. 5. Examples of topological transformations

In the iterative process depicted in Fig. 5, Stages 1-8 demonstrate shape optimization facilitated by the movement of boundary nodes. Between iterations 8 and 9, a topological change is triggered through material flipping when boundary elements meet specific criteria. In iterations 9 to 10, it is evident that the distorted mesh left after the material flip is smoothed out in the following iteration, facilitating a more natural transition of the overall mesh configuration.

This sequence effectively shows how shape and topology adjustments are accomplished in structural optimization by combining node movement, material flipping, and mesh smoothing. This comprehensive approach not only optimizes structural designs for enhanced performance and efficiency but also progressively refines the shape and structure of the mesh.

4. Test results of the BMM method for 2D classic benchmark topological optimization problems

This section examines some of the classic problems in topology optimization for evaluation purposes, including three-point bending beams, L-shaped beams, cantilever beams, simply supported beams, two-force members, and quarter-circle arches. The last four are presented in the appendix. The optimization outcomes are compared with results obtained via the SIMP method and the BESO method.

(1) Simply supported beam problem

Figure 6 displays the optimization results for the simply supported beam divided into a 60×30 mesh grid, where $\rho = 50\%$. The initial configuration features a rectangular structure with an array of square holes. As the optimization iterations progress, adjustments occur at the hole edges: contraction in areas of higher strain energy and expansion in regions of lower strain energy. By the 50th iteration, the boundary elements of some holes meet the critical condition for topological change. As the number of iterations increases, numerous holes begin to merge, leading to significant topological transformations and gradually forming the preliminary shape of the optimized structure. By the 470th iteration, the fragmented branches have been smoothed out, resulting in a more streamlined overall structure.

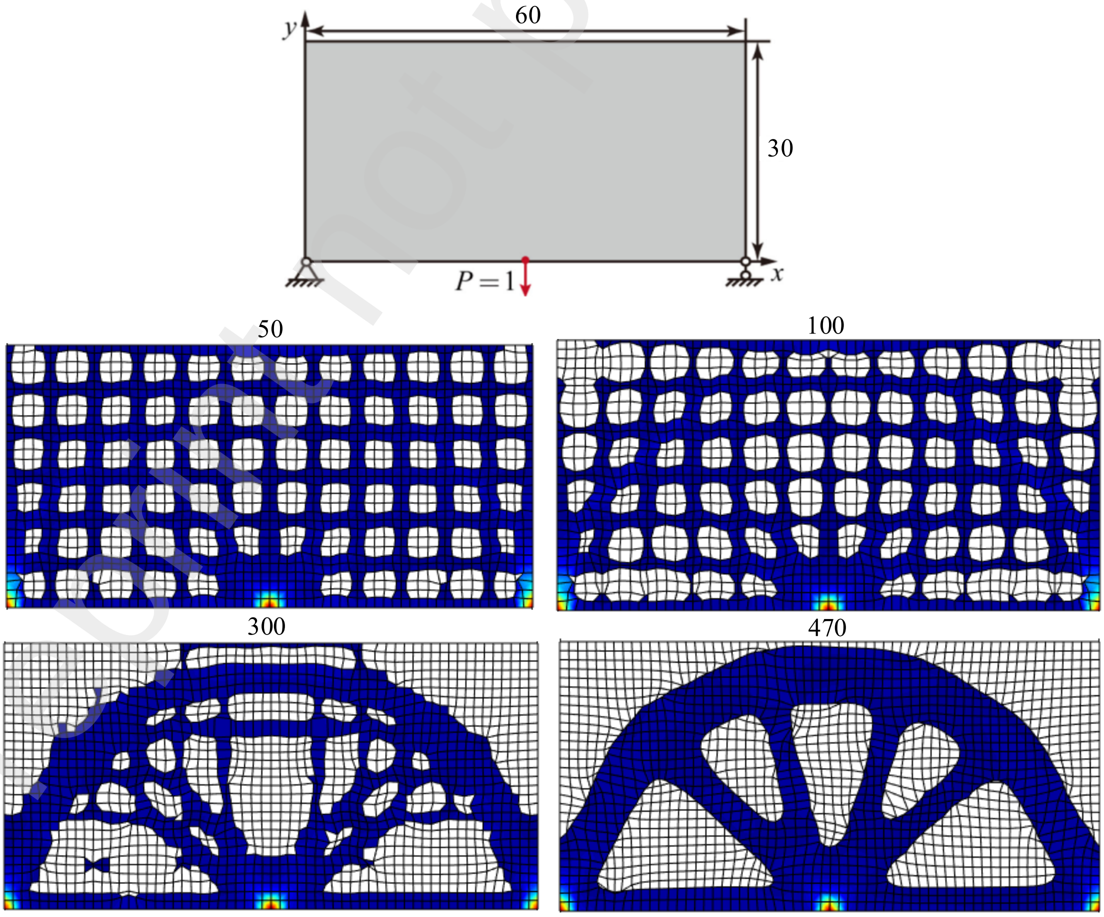


Fig. 6. Iterative process of the BMM method for a simply supported beam problem.

The use of movable nodes in the BMM method allows for smoother edges, improving the geometric representation. In contrast, traditional optimization methods often produce jagged and rough mesh models, which require additional steps—such as geometric feature extraction, boundary smoothing, and mesh redivision—before implementation. This advantage of the BMM method streamlines the application of optimization results, enhancing efficiency and reducing manual intervention, making the outcomes more immediately applicable to practical engineering projects.

(2) L-beam problem

By comparing the optimization results of the L-shaped beam divided into a 60×60 mesh grid in Fig. 7 with $\rho = 50\%$, we can discern the performance characteristics of various optimization methods. The L-shaped beam optimized by the BMM method features an inclined edge at the left support, demonstrating a degree of geometric complexity. In contrast, while the BESO method also attempts to depict sloped features, the limitations of the fixed grid can result in a disjointed appearance. Similarly, the SIMP method, constrained by its fixed grid, cannot smoothly represent sloped edges; instead, it indirectly suggests this feature through variations in color blocks.

Although methods using fixed grids can address this issue by employing finer mesh divisions, under the same grid count, methods utilizing movable grids exhibit a distinct advantage. This example underscores the BMM method's superior capability in geometric representation, which is particularly effective in handling complex structures such as inclined edges and curves.

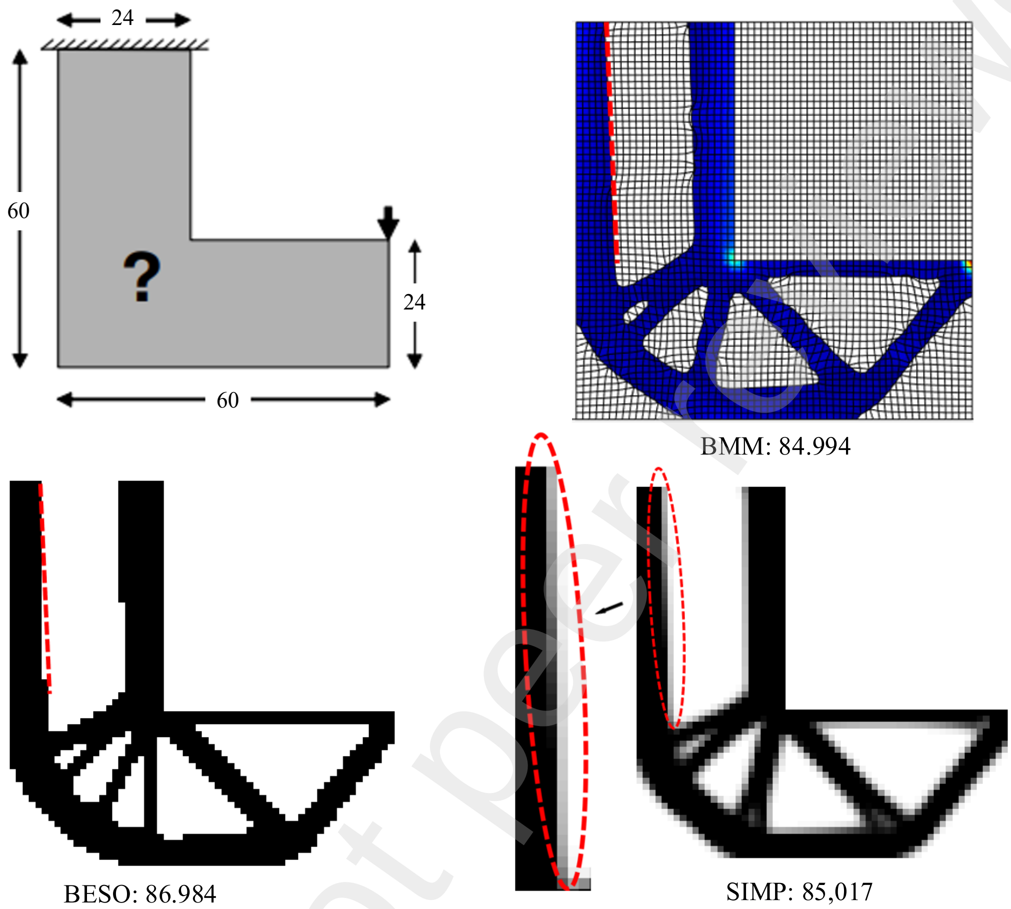


Fig. 7. Comparing the optimization results of different TO methods for the L-beam problem.

5. Application of the BMM method in 3D topology optimization

In three-dimensional settings, the node-driven and material element-flipping methods remain applicable. Materials can be flipped on the basis of criteria such as the volume, aspect ratio, and concavity of the element. A detailed description is as follows:

1. Boundary motion

Similarly, the assumption that the nodal displacement is linearly related to sensitivity can also be applied. The only difference is that in three dimensions, the normal vector of contour nodes is more difficult to define. The direction of the vector can be represented by the vector average of the contour node and the centroids of the surrounding solid elements.

2. Material Flipping

For elements with a volume less than 0.2 or nonconvex elements, the material of the element should be flipped.

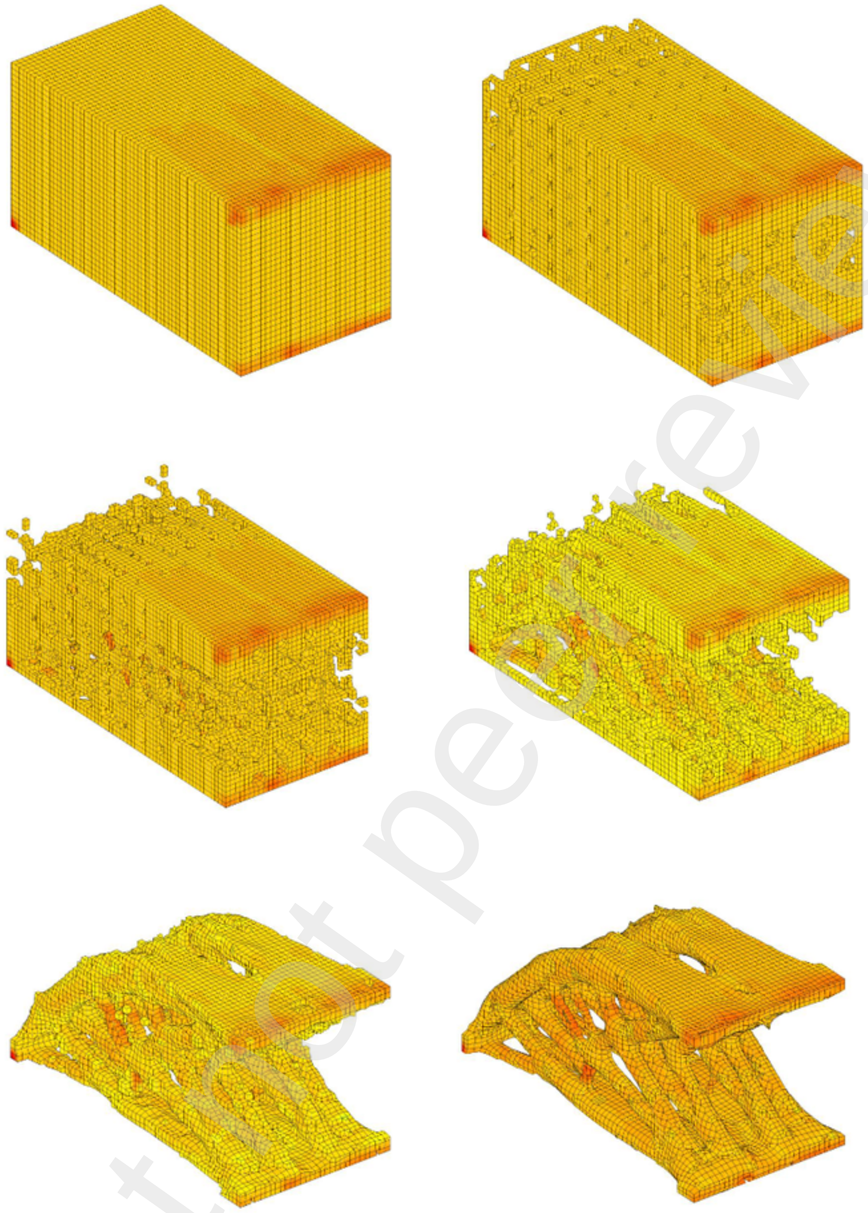
3. Mesh smoothing

The homogenization of internal nodes can still be calculated via the average of all adjacent nodes of the internal node.

Figure 8 illustrates an example of applying the BMM method in a 3D model. Within a 60×30×30 mesh design domain, one side is fixed, and a downward force is applied on the opposite side with a volume fraction constraint of 20%.

This demonstrates that the BMM method excels not only in 2D scenarios but also in optimizing 3D structures. Employing node-driven and material element flipping strategies, the BMM method effectively optimizes complex 3D structures, yielding more efficient and superior outcomes. The implementation of this method expands the options and possibilities for engineering design and optimization, providing valuable solutions to the challenges associated with complex structural optimization. The result exhibits a well-defined contour, in contrast to the SIMP method, which can only optimize an abstract structure with noticeable mosaic jagged edges for the same mesh size. Furthermore, the objective function of the BMM method shows a 24% improvement over that of the SIMP method.

(a) The iterative process of the BMM Method



(b) Comparison of the results between the BMM method and SIMP method

BMM method: 2376.2 SIMP method: 3109.3

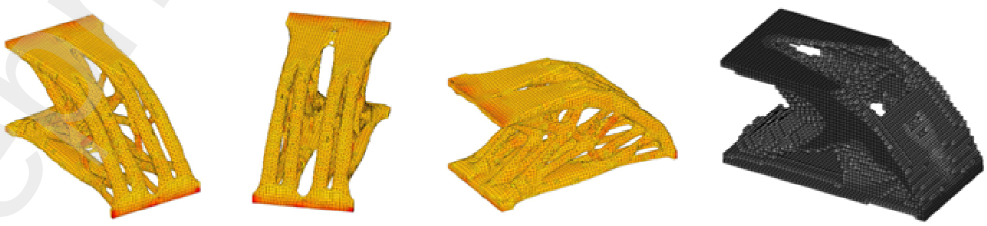


Fig. 8. Comparing the optimization results between the BMM and SIMP methods for the 3D cantilever beam problem.

With an equivalent number of meshes, the BMM method takes 63 seconds per iteration, whereas the SIMP method takes 35 seconds per iteration because the BMM method sacrifices time in assembling the stiffness matrix for different shaped elements to achieve effects that would require a finer mesh and more time consumption in classic methods. Moreover, other topology optimization methods are similar to direct methods for solving linear equations, whereas the BMM method, with its local calculation, resembles iterative methods, such as multigrid methods and multilevel subspace methods, which are well suited for large-scale parallel computations. When the scale of optimization is significantly large, the BMM method has more significant advantages over traditional approaches such as the SIMP method.

6. Conclusions and discussion

In this paper, we propose the biomimetic moving-mesh (BMM) topology optimization method inspired by biological cell growth and evolution. The following conclusions can be drawn.

1. The BMM method can solve the boundary nonsmoothness problem encountered in traditional pixel–point topology optimization methods, with optimized geometric descriptions consistent with those of finite element analysis.

2. Through many case studies, it has been found that the BMM achieves better results than the SIMP and BESO methods with comparable numbers of meshes due to the variability of the mesh. Alternatively, the BMM can achieve equivalent results to those of the other methods with fewer meshes.

3. The BMM method is a locally adaptive optimization method that is particularly suitable for parallel large-scale optimization compared with the overall optimization class of methods.

Moreover, the BMM method encounters challenges such as prolonged stiffness matrix formation times, the ability to handle complex sharp geometries, and the stability of the optimization pathway. These issues can be addressed through combined optimization via multiple methods. For example, initial optimization with methods such as SIMP, BESO, MMC, or the level set method can establish the basic topology, followed by detailed boundary adjustments via the BMM method to increase boundary smoothness and material utilization. This integrated approach capitalizes on the strengths and mitigates the weaknesses of various topology optimization techniques.

To facilitate a better understanding of the BMM implementation process, a MATLAB code example of the BMM algorithm is provided in the supplementary material.

Appendix

In the appendix, four more examples provide further evidence of the successful application of the BMM method.

MBB problem

The MBB problem is also a classic problem in topology optimization where both ends are simply supported, and a downward concentrated force is applied at the center from above.

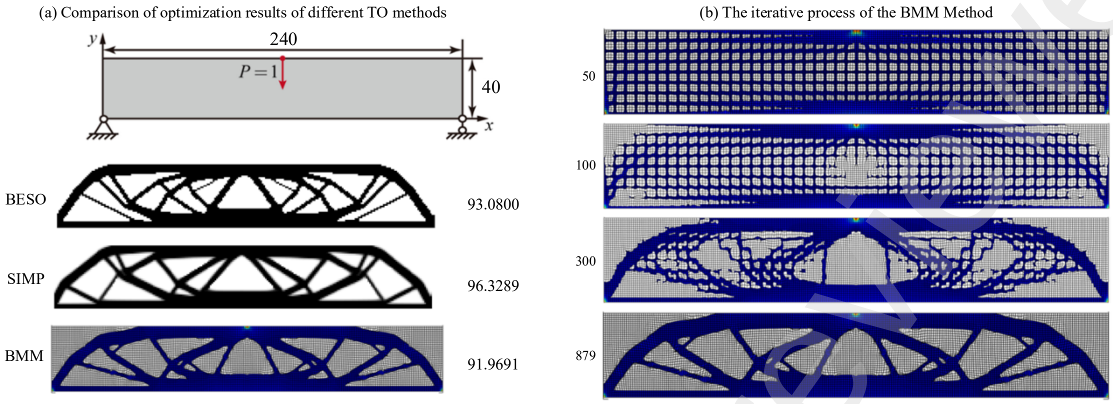


Fig. 9. Comparing the optimization results of different TO methods for the MBB problem.

With a volume fraction constraint of 50%, the BMM method achieves superior outcomes compared with both the BESO and SIMP methods for an identical 240×40 grid size, as illustrated in Fig. 9.

Cantilever problem

The cantilever beam problem is a classic challenge in topology optimization where the left end is fixedly supported and the right end is subjected to a downward concentrated force.

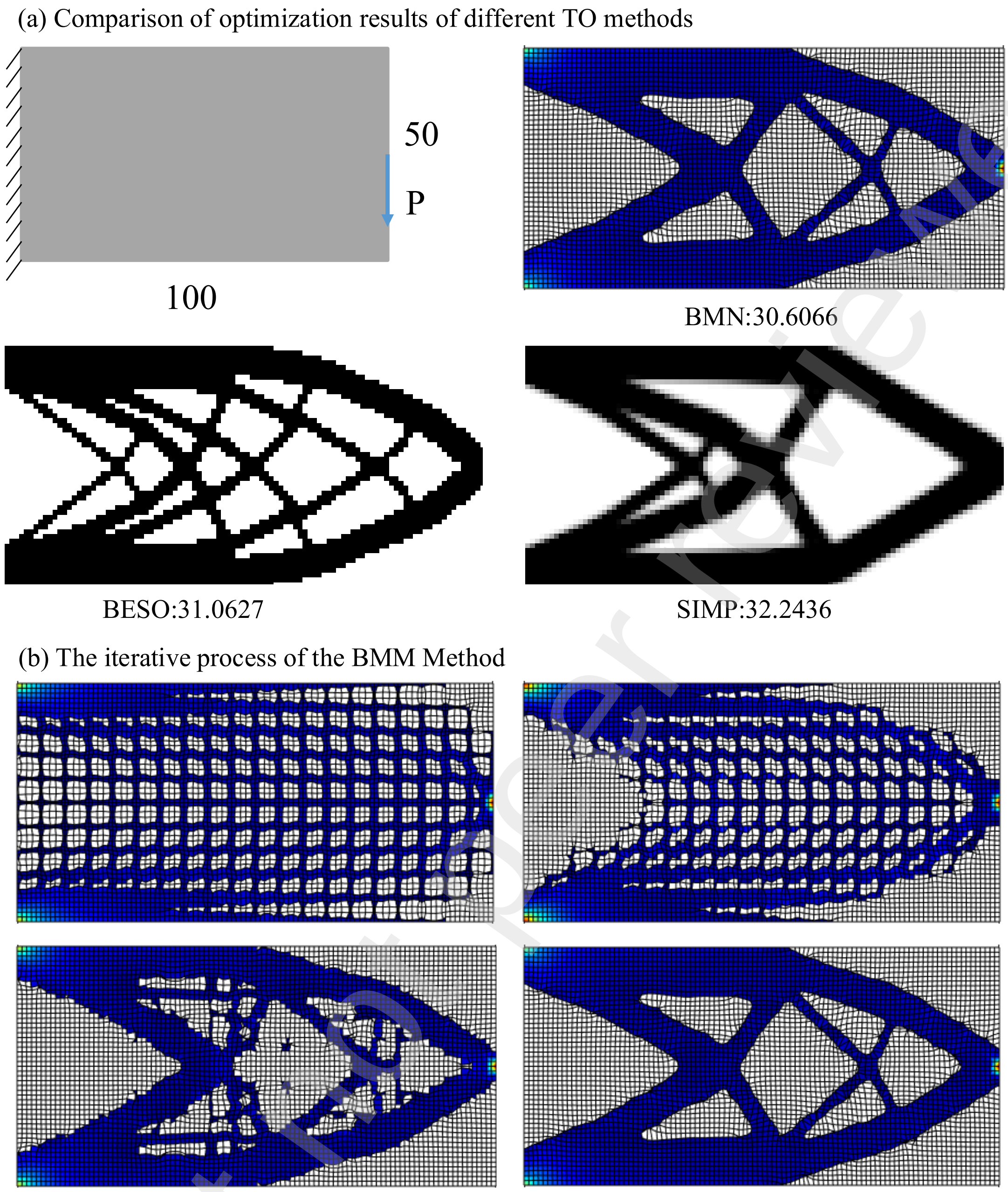


Fig. 10. Comparing the optimization results of different TO methods for the cantilever problem.

With a volume fraction constraint of 50%, the BMM method achieves superior outcomes compared with both the BESO and SIMP methods for an identical 100×50 grid size, as illustrated in Fig. 10.

Quarter annulus problem

In this example, the design domain of a quarter annulus is segmented into annular meshes, establishing an initial array of periodic holes. The optimization process, illustrated in Fig. 11, shows that grids with curved boundaries can be effectively implemented within the BMM model. Owing to the dynamic grid properties of the BMM method, the design domain is not restricted to fixed grid types and can be customized to align more closely with the structural characteristics and requirements of the study.

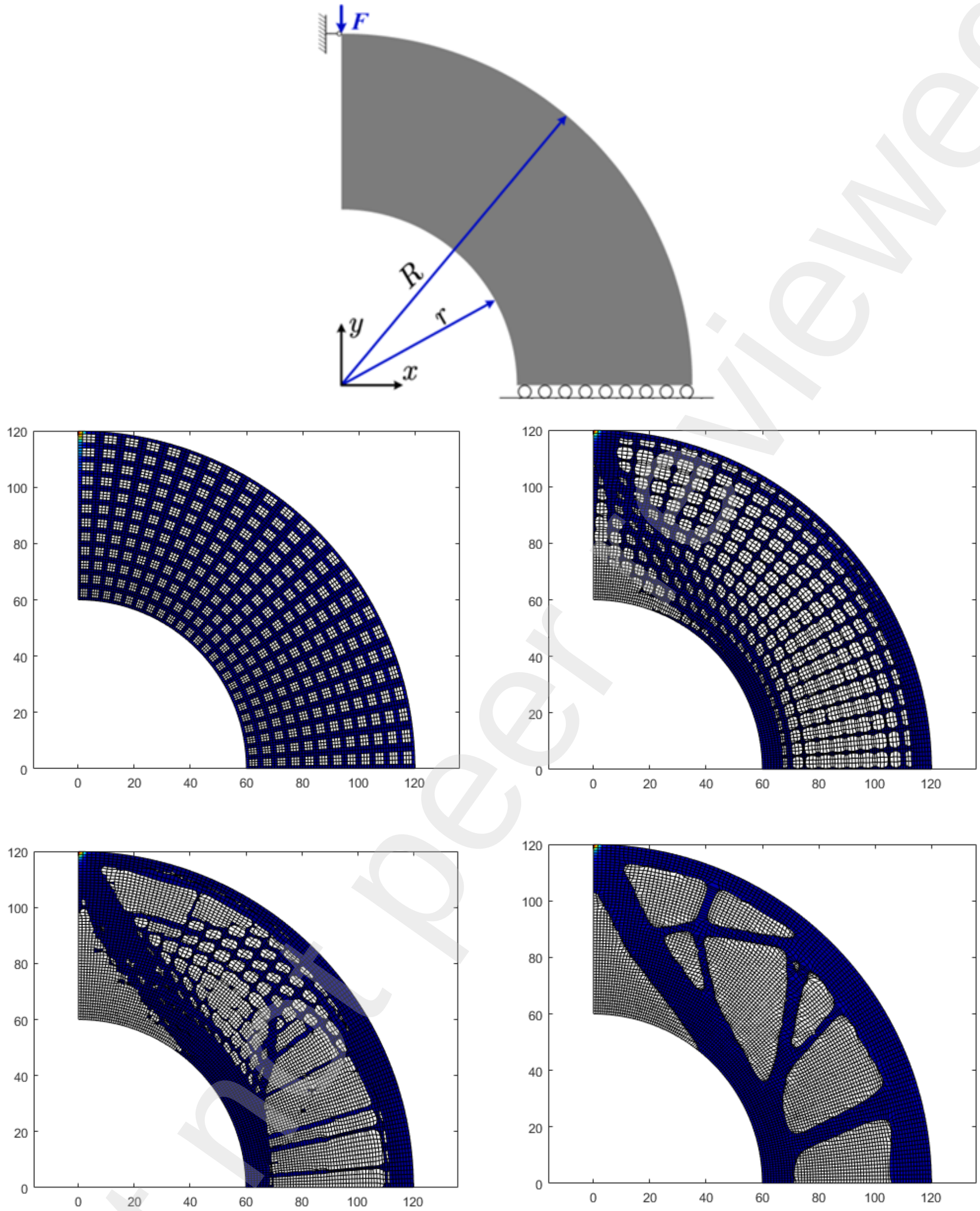


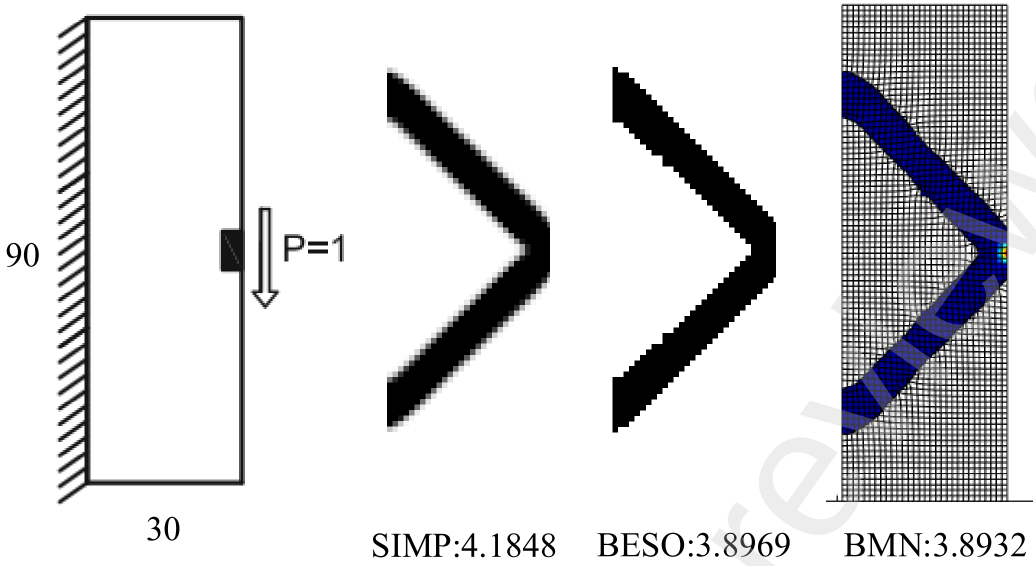
Fig. 11. The iterative process of the BMM method for the quarter annulus problem.

Other classic topological optimization problems, including the cantilever beam and the special cantilever beam with an optimal analytical solution, have also yielded superior results using the BMM method compared with traditional topological optimization approaches.

Special cantilever beam problems with optimal analytic solutions

In this specific cantilever beam problem, the design domain is a 3:1 rectangle with a downward concentrated force applied at the midpoint of the right end. This configuration has an optimal analytical solution consisting of two two-force rods forming a 90-degree angle. According to the results depicted in Fig. 12, when a discretized 30 × 90 mesh with a volume fraction constraint of 20% is used, the BMM, BESO, and SIMP methods are all capable of achieving the standard optimal structure. However, the BMM method demonstrates slight superiority in terms of the objective function.

(a) Comparison of optimization results of different TO methods



(b) The iterative process of the BMM Method

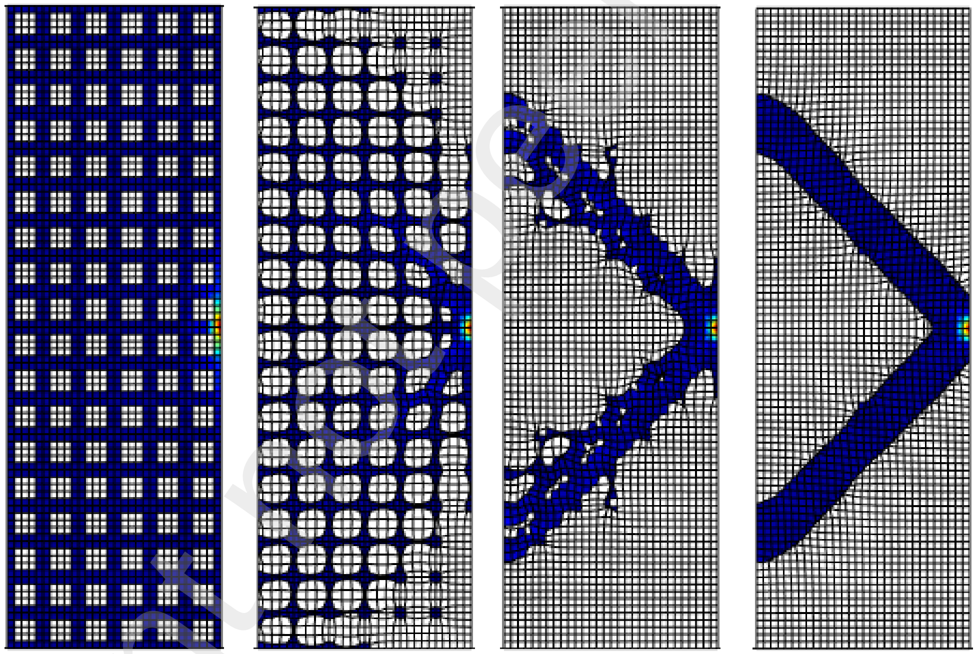


Fig. 12. Comparing the optimization results of different TO methods for the special cantilever beam problem with optimal analytic solutions.

Notably, the results from the BMM method exhibit rounded transition details at the corners, whereas the SIMP method requires a mesh refinement of up to 200×600 to achieve comparable results. This case demonstrates that the BMM method achieves equivalent optimization outcomes with fewer meshes than other classic topology optimization methods do.

CRediT authorship contribution statement

Huawei Feng: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Writing – original draft, Writing – review & editing, Visualization. **Zhongqi Li, Peidong Lei, Junjie Zhou:** Formal analysis, Writing – review & editing, Visualization. **Huikai Zhang:** Conceptualization, Formal analysis, Writing – review & editing. **Bin Liu:** Conceptualization, Formal analysis, Writing – review & editing, Supervision, Funding acquisition.

Declaration of competing interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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